Heat Conduction and Fluid Flow in Porous Media

The temperature distribution in an aquifer is influenced by the interplay of heat conduction and advective flow. In section 3.4 of the text this is illustrated by an example for the steady-state flow of heat and fluid in the vertical direction.

- **Heat conduction** is described by **Fourier's law** which relates the *conductive heat flux* q_h to *thermal gradient* $\frac{dT}{dz}$:

$$q_h = -K_m \frac{dT}{dz} \tag{1}$$

  where K_m is the *thermal conductivity*. SI units for the flux q_h and conductivity K_m are W/m^2 (J/s/m^2) and W/(m-K), respectively. Typical thermal conductivy values are tabulated below (after *Deming*).

Material	Thermal Conductivity (W/(m-K)
diamond	1489
copper	385
water	0.6
air	0.024
rocks and minerals	1 - 7
shale	1 - 2
sandstone	2.0 - 4.5
limestone	2 - 4
granite	3 - 4
basalt	1.5 - 2.5
coal	0.2 - 0.4

- For **1-dimensional** groundwater flow in the *vertical* direction, the governing equation as given by equation (3.18) can be simplied to

$$\rho_b C_b \frac{\partial T}{\partial t} = K_m \frac{\partial^2 T}{\partial z^2} - q_z \rho_w C_w \frac{\partial T}{\partial z} + R_h \tag{2a}$$

  where ρ_b and ρ_w denote the *densities of the saturated rock and water*, C_b and C_w denote the *specific heat capacities* of saturated rock and water, q_z is the specific discharge (Darcy velocity) in the vertical direction, and R_h is *heat generation rate*. SI units for specific heat and heat generation rate are J/(kg-K) and W/m^3 (J/m^3/s). (The saturated rock density and specific heat can also be expressed as $\rho_b C_b = n\rho_w C_w + (1-n)\rho_r C_r$ where n is the porosity, ρ_r is the density of the solid grain, and C_r is the specific heat of the solid grain.) In this application the heat production (which sarise primarily from radioactive heat) is assumed to be negligible, so $R_h = 0$. The first term that represents transient temperature changes is assumed to be negligible for **steady state**, so we are left with the following equation

$$K_m \frac{d^2 T}{dz^2} - q_z \rho_w C_w \frac{dT}{dz} = 0 \tag{2b}$$

- An analytic steady-state solution for this problem was derived by *Bredehoeft and Papadopolous* (1965). If the temperatures of the upper and lower boundaries are denoted by T_U and T_L, and the distance between the two constant-temperature boundaries are denoted by L, then the temperature $T(z)$ is given by equation (3.19) of the text

$$T(z) = T_U + (T_L - T_U)\frac{\left[\exp(\beta\frac{z}{L}) - 1\right]}{[\exp(\beta) - 1]} \tag{3}$$

where $\beta = (q_z \rho_w C_w L)/K_m$ corresponds to the **Peclet number** for this setting, a dimensionless quantity that characterizes the relative magnitude of convective to conductive transport. If convective (advective) transport is negligible, then $q_z = 0$ and consequently $\beta = 0$. The function $f(\beta, \frac{z}{L}) = \frac{\left[\exp(\beta\frac{z}{L})-1\right]}{[\exp(\beta)-1]}$ is plotted below to illustrate the effect of upward and downward fluid flow (corresponding to positive and negative values of the Peclet number) on temperature as a function of depth.